Microscopic Cluster Model for Exotic Nuclei

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Abstract

For a better understanding of the dynamics of exotic nuclei it is of crucial importance to develop a practical microscopic theory easy to be applied to a wide range of masses. Theoretically the basic task consists in formulating an easy solvable theory able to reproduce structures and transitions of known nuclei which should be then used to calculate the sparely known properties of proton- or neutron-rich nuclei. In this paper we start by calculating energies and distributions of $A \leq 4$ nuclei withing a unitary correlation model restricted to include only two-body correlations. The structure of complex nuclei is then calculated extending the model to include correlation effects of higher order.

1 Introduction

The experimental efforts presently performed at the new generation of radioactive ion beam facilities in particular the Rare Isopote Accelerator (RIA) in USA, the international accelerator facility for research with anti-proton ions at GSI in Germany, and the Radioactive Ion Beam Factory (RIBF) in Japan, will boost the nuclear structure studies of exotic nuclei.

One of the central challenges of theoretical nuclear physics is the attempt to describe these unknown properties of the exotic systems in terms of a realistic nucleon-nucleon (NN) interaction. In order to calculate matrix elements with this singular interaction we have to define effective correlated Hamiltonians.

Correlation effects in nuclei have been first introduced in nuclei by Villars [1], who proposed the unitary-model operator (UMO) to construct effective operators. The method was implemented by Shakin [2] for the calculation of the G-matrix from hard-core interactions. Non perturbative approximations of the UMO have been recently applied to odd nuclei in Ref. [3] and to even nuclei in Ref. [4]. The basic formulas of the Dynamic Correlation Model and of the Boson Dynamic Correlation Model (BDCM) presented in the above quoted papers have been obtained by separating the n-body correlation operator in short- and long-range components. The short-range component is considered up to the two body correlation while for the long range component the three and four body correlation operators have been studied. The extension of the correlation operator to high order diagrams is especially important in the description of exotic nuclei (open shell). In the short range approximation the model space of two interacting particles is separated in two subspaces: one which includes the shell model states and the other (high momentum) which is used to compute the G-matrix of the model. The long range component of the correlation operator has the effect of generating a new correlated model space

(effective space) which departs from the originally adopted one (shell model). The amplitudes of the model wave functions are calculated in terms of non linear equation of motions (EoM). The derived systems of commutator equations, which characterize the EoM, are finally linearized. Within these generalized linearization approximations (GLA) we include in the calculation presented in the paper up to the ((n+1)p1h) effective diagrams. The linearized terms provide, as explained later in the text, the additional matrix elements that convert the perturbative UMO expansion in an eigenvalue equation. The n-body matrix elements needed to diagonalize the resulting eigenvalue equations are calculated exactly via the Cluster Factorization Theory (CFT).

Within the present treatment of the correlation operator one generates in the n-body theory not only the ladder diagrams of Ref. [6] but also the folded diagrams of Kuo see Ref. [7].

2 The Two-body Effective Interaction

In order to describe the structures and the distributions of nuclei we start from the following Hamiltonian:

$$H = \sum_{\alpha\beta} \langle \alpha|t|\beta\rangle \, a^{\dagger}_{\alpha} a_{\beta} \; + \; \sum_{\alpha\beta\gamma\delta} \langle \Phi_{\alpha\beta}|v_{12}|\Phi_{\gamma\delta}\rangle \, a^{\dagger}_{\alpha} a^{\dagger}_{\beta} a_{\delta} a_{\gamma} \tag{1}$$

where v_{12} is the singular nucleon-nucleon two body potential. Since the matrix elements $|\alpha\beta\rangle$ are uncorrelated the matrix elements of v_{12} are infinite. This problem can be avoided by taking matrix elements of the Hamiltonian between correlated states. In this paper the effect of correlation is introduced via the e^{iS} method. In dealing with very short range correlations only the S_2 part of the correlation operator needs to be considered.

Following Ref. [2] we therefore calculate an "effective Hamiltonian" by using only the S_2 correlation operator obtaining:

$$\begin{array}{l} H_{eff} = e^{-iS_2} H e^{iS_2} = \sum_{\alpha\beta} \langle \alpha|t|\beta\rangle a^{\dagger}_{\alpha} a_{\beta} + \sum_{\alpha\beta\gamma\delta} \langle \Psi_{\alpha\beta}|v^{l}_{12}|\Psi_{\gamma\delta}\rangle a^{\dagger}_{\alpha} a^{\dagger}_{\beta} a_{\delta} a_{\gamma} \\ = \sum_{\alpha\beta} \langle \alpha|t|\beta\rangle a^{\dagger}_{\alpha} a_{\beta} + \sum_{\alpha\beta\gamma\delta} \langle \Psi_{\alpha\beta}|v|\Psi_{\gamma\delta}\rangle a^{\dagger}_{\alpha} a^{\dagger}_{\beta} a_{\delta} a_{\gamma} \end{array} \tag{2}$$

where v^{l}_{12} refers to the long-range part of the nucleon-nucleon force diagonal in the relative orbital angular momentum, after the separation [8]:

$$v_{12} = v^{s}_{12} + v^{l}_{12} \tag{3}$$

The separation is made in such a way that the short range part produces no energy shift in the pair state [8]. In doing shell model calculation with the Hamiltonian Eq. (2), we remark: a) only the long tail potential plays an essential role in the calculations of the nuclear structure i.e.: the separation method and the new proposed v_{low-k} [9] method show a strong analogy and b) the v^{od}_{T} must be included as an additional re-normalization of the effective Hamiltonian Eq. (2).

In Eq. (2) the $\Psi_{\alpha\beta}$ is the two particle correlated wave function:

$$\Psi_{\alpha\beta} = e^{iS_2} \Phi_{\alpha\beta} \tag{4}$$

In order to evaluate the effect of the tensor force on the $\Psi_{\alpha\beta}$ we calculate:

$$w(r) = V^{od}_{T} \frac{Q}{\Delta E} u(r) = V^{od}_{T} \frac{Q}{\Delta E} |(\tilde{nl}S), J' : NL, J\rangle \tag{5}$$

where Q is a momentum dependent projection operator, $\Delta E(k_1, k_2)$ the energy denominator and $\tilde{nl}$ the correlated two particle state in the relative coordinates. In Eq. (5) u(r) is generated as in Ref. [2] by

a separation distance calculation for the central part of the force in the $^3S_1$ state. The wave function obtained in this way (full line) heals to the harmonic-oscillator wave function (dashed line) as shown in Fig. 1. The result obtained for Eq. (5) calculated with the tensor force of the Yale potential [10] is given also in Fig. 1 left where we plot for the harmonic oscillator size parameter b=1.41 fm:

$$\Psi(\vec{r}) = [u(r)Y_0^1(\Omega_{\vec{r}}) + w(r)Y_2^1(\Omega_{\vec{r}})] \tag{6}$$

Being the admixture of the two components, circa 4%, the wave function Eq. (6) can be associated to the deuteron wave function.

Let us use the the Hamiltonian Eq. (2) to calculate the structure of the A=3 nuclei. Here we propose to calculate the ground state of ^{3}H, ^{3}He, and ^{4}He within the EoM method which derive the eigenvalue equations by working with the e^{iS_2} operator on the wave functions of the A=3,4 nuclei.

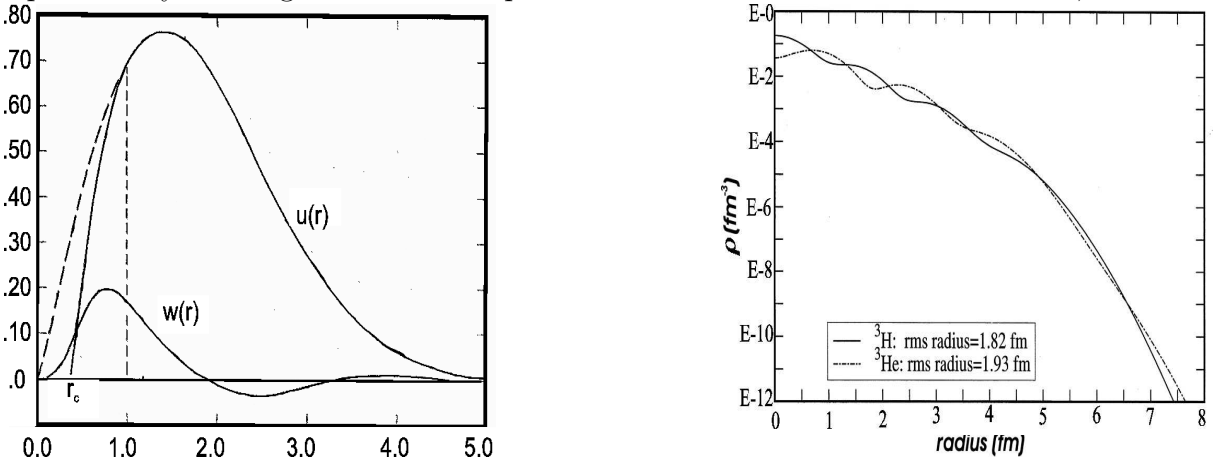

Figure 1: Left: The u(r) and w(r) wave functions of the deuteron, with quantum numbers $^3S_1$ and $^3D_1$, plotted as function of r; Right: Distributions of ^{3}H and ^{3}He.

3 The Few-body Problem.

We write the three particle states in second quantization by discarding for simplicity the isospin quantum numbers:

$$\Phi_{3p} \longrightarrow A_1^\dagger(\alpha_1 J_1 J)|0\rangle = [a_{j_1}^\dagger (a_{j_2}^\dagger a_{j_3}^\dagger)^{J_1}]_M^J |0\rangle, \tag{7}$$

where the operators $a_{j_1}^\dagger a_{j_2}^\dagger a_{j_3}^\dagger$ create three particles in the open shells and we analyze the structure of the particle dynamics, generated by the correlation operator, via the following commutator:

$$[H, A_1^\dagger(\alpha_1 J_1 J)]|0\rangle = [(\sum_\alpha \epsilon_\alpha a_\alpha^\dagger a_\alpha + \frac{1}{2}\sum_{\alpha\beta\gamma\delta}\langle\alpha\beta|v(r)|\gamma\delta\rangle a_\alpha^\dagger a_\beta^\dagger a_\delta a_\gamma), (a_{j_1}^\dagger (a_{j_2}^\dagger a_{j_3}^\dagger)^{J_1})^J]|0\rangle. \tag{8}$$

By taking the expectation value of the commutator Eq. (8) between the vacuum and the three particle states we obtain the eigenvalue equation. In order to solve Eq. (8) we introduce the CFT [5] which factorizes the three particle states in combination of pairs. For the two particles we distinguish between two spaces: 1) effective valence space which is used to diagonalize the EoM and 2) complementary high excited single particle space which is used to compute the G matrix. Within this method we can use either the Shakin-Yale matrix elements of Ref. [11] with b=1.50 fm or the V_{low-k} matrix elements [9]. From the diagonalization of the eigenvalue equation of the three particles, we obtain an energy difference $\Delta E(^3H - ^3He)$=0.78 MeV and the charge distributions and radii given in Fig. 1 Right. By extending the

commutator of Eq. (8) to a four particle state we obtain for the ground state of ^{4}He the binding energy of E=28.39 Mev. From the the ground state wave functions calculated with the two-body potential of Ref. [11] (b=1.50 fm) and with the V_{low-k} matrix elements [9] evaluated with the Bonn potential [12] we obtain the two distributions given in Fig. 2) left. The distributions and the radii of 1.709 fm and 1.71 fm look similar.

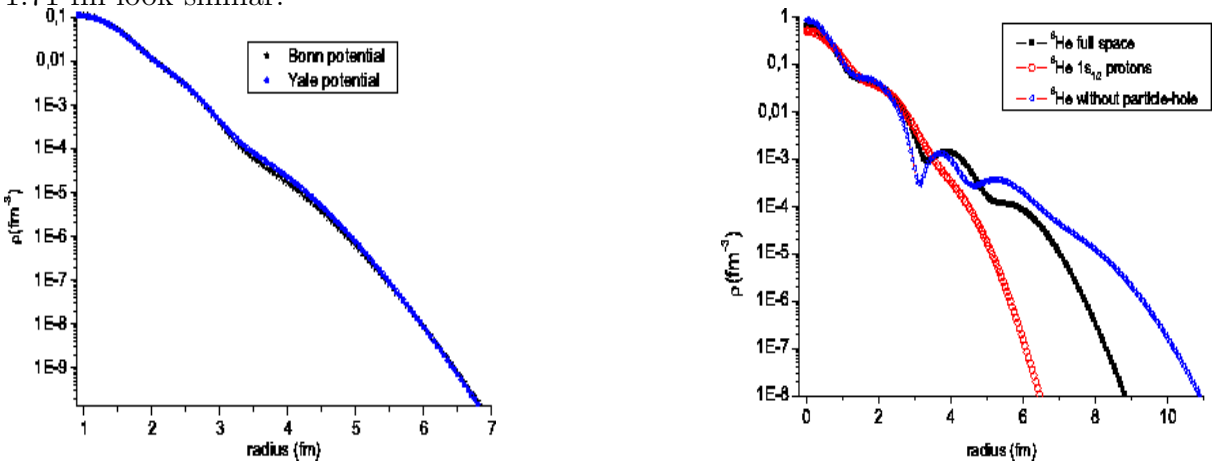

Figure 2: Left: Distributions of ^{4}He calculated with the Shakin-Yale and the V_{low-k} potentials; Right: Charge distributions of ^{6}He calculated in different approximations.

4 The n-Body Problem

In dealing with complex nuclei however the $(S_i,\ i = 3 \cdots n)$ correlations should also be considered. The evaluation of these diagrams is, due to the exponentially increasing number of terms, difficult in a perturbation theory. We note however that one way to overcome this problem is to work with $e^{i(S_1+S_2+S_3+\cdots+S_i)}$ operator on the Slater's determinant by keeping the n-body Hamiltonian uncorrelated. Via the long tail of the nuclear potential the Slater determinant of the "n" particle systems are interacting with the excited Slater's determinants formed by the ("n" particles+(mp-mh) mixed-mode excitations). The amplitudes of the different determinants are calculated via the EoM method.

After having performed the diagonalization of the n-body Hamilton's operator we can calculate the form of the effective Hamiltonian which, by now, includes the complete set of the commutator equations. For odd particle systems we use the well established formalism of Ref. [3].

4.1 Two "dressed" Particles

In the following we present the theory for even "dressed" particles. In order to define the eigenvalue equations for the nuclear modes we start by defining the two particle operator as following:

$$\Phi_{2p} \longrightarrow A_1^\dagger(\alpha_1 J)|0\rangle = [a_{j_1}^\dagger a_{j_2}^\dagger]_M^J|0\rangle, \tag{9}$$

We calculate then the commutator equations:

$$[H, A_1^\dagger(\alpha_1 J)]|0\rangle = \sum_{\beta_1} \Omega(2p|2p')A_1^\dagger(\beta_1 J)|0\rangle + \sum_{\beta_2 J_1' J_2'} \Omega(2p|3p1h)A_2^\dagger(\beta_2 J_1' J_2' J)|0\rangle. \tag{10}$$

and

$$\begin{aligned}&[H, A_2^\dagger(\alpha_2 J_1 J_2 J)]|0\rangle \\ &= \sum_{\beta_2 J_1' J_2'} \Omega(3p1h|3p'1h')A_2^\dagger(\beta_2 J_1' J_2' J)|0\rangle + \sum_{\beta_3 J_1' J_2' J_3'} \Omega(3p1h|4p2h)A_3^\dagger(\beta_3 J_1' J_2' J_3' J)|0\rangle,\end{aligned} \tag{11}$$

which define the dynamic evolution of the valence modes. In Eqs. (10,11) the $A_2^\dagger(\beta_2 J_1' J_2' J)$ operators are defined below:

$$\Phi_{3p1h} \longrightarrow A_2^\dagger(\beta_2 J_1' J_2' J)|0\rangle = ((a_{j_1'}^\dagger a_{j_2'}^\dagger)^{J_1'}(a_{j_3'}^\dagger a_{j_4'})^{J_2'})^J|0\rangle. \tag{12}$$

The additional commutator equations which involves the $A_3^\dagger(\beta_3 J_1' J_2' J_3' J)$ and the higher order operators are here not given. The obtained commutator chain is suitable to be solved perturbatively by inserting the n-th commutator in the $(n-1)$-th commutator, $(n-1)$-th commutator in the $(n-2)$-th commutator , ... the second commutator in the first. Within this perturbative approach one defines effective Hamiltonians of the model which, due to the increasing degree of complexity, are not easily solvable. Much simpler solutions to the commutator equations may, however, be obtained in the BDCM model. We start by remarking that in the study of low lying excitations of the n-body systems the higher order components of the wave functions, which involve n valence - and (2p-2h) core-excitations are poorly admixed in the model space and can be linearized. This linearization consists by applying the Wick's theorem to the $A_3^\dagger(\beta_3 J_1' J_2' J_3' J)$ terms and by neglecting the normal order diagrams. The linearization generates the additional terms that one needs to convert the commutator chain in the corresponding eigenvalue equation, as can be obtained by taking the expectation value of the linearized Eqs. (10), and (11) between the vacuum and the model states.

4.2 Three "dressed" Particles

For three "dressed" particles we extend the commutator equation of Eq. (7) obtaining the following non-linear commutator equations:

$$[H, A_1^\dagger(\alpha_1 J_1 J)]|0\rangle = \sum_{\beta_1} \Omega(3p|3p')A_1^\dagger(\beta_1 J_1')|0\rangle + \sum_{\beta_2 J_1' J_2'} \Omega(3p|4p1h)A_2^\dagger(\beta_2 J_1' J_2' J)|0\rangle. \tag{13}$$

and

$$\begin{aligned}&[H, A_2^\dagger(\alpha_2 J_1 J_2 J)]|0\rangle \\ &= \sum_{\beta_2 J_1' J_2'} \Omega(3p|3p')A_2^\dagger(\beta_2 J_1' J_2' J)|0\rangle + \sum_{\beta_3 J_1' J_2' J_3'} \Omega(3p|4p2h)A_3^\dagger(\beta_3 J_1' J_2' J_3' J)|0\rangle,\end{aligned} \tag{14}$$

In Eqs. (13,14) the $A_2^\dagger(\beta_2 J_1' J_2' J)$ operators are defined below:

$$\Phi_{3p1h} \longrightarrow A_2^\dagger(\beta_2 J_1' J_2' J)|0\rangle = ((a_{j_1'}^\dagger (a_{j_2'}^\dagger a_{j_3'}^\dagger)^{J_1'})^{J_i'}(a_{j_5'}^\dagger a_{j_4'})^{J_2'})^J|0\rangle. \tag{15}$$

The additional commutator equations are here not given. The ($A_3^\dagger(\beta_3 J_1' J_2' J_3' J)$ (4p1h) terms in Eq. (15) are finally linearized. By neglecting the normal order terms we obtaing the terms which convert the commutator chain in coupled non-linear equations for three particles interanting with the (4p1h) states. The eigenvalue equation for the three dressed particles is then obtained by taking the expectation value of the linearized Eqs. (13), and (14) between the vacuum and the excited states.

The generalization of the commutator equations to "n" valence particles can be simply derived in the analogous way and is not given. Within these approximations the model commutator equations are suitable to be restricted to a finite space. The linearized system of the commutator equations is then solved exactly in terms of the CFT [5] which calculates the n-body matrix elements of one- and two-body operators.

5 Results

In order to perform structure calculations, we have to define the CMWFs base, the "single-particle energies" and to choose the nuclear two-body interactions. The CMWFs are defined as shown in Appendix

	DCM	Exp. [16]
Magnetic Moment (mm)	.70	.7189
	DCM	Exp. [17]
rms (fm)	2.74	2.73(3)

Table 1: Magnetic moment (nm) and rms (fm) of the ground state of ^{15}O with $J = \frac{1}{2}^{-}; T = \frac{1}{2}$

Energy (MeV)	DCM	Exp. [16]
$\Delta E_{\frac{1}{2}^- \frac{5}{2}^+}$	5.41	5.24
Ratio	DCM	Exp. [16]
$\frac{BE(E3;\frac{5}{2}^+ \rightarrow \frac{1}{2}^-)}{BE(M2;\frac{5}{2}^+ \rightarrow \frac{1}{2}^-)}$	.15	.10

Table 2: Energy splitting between the ground- and the first excited state and the corresponding electromagnetic transitions for ^{15}O.

A by including mixed valence modes and core-excited states. The base is then orthonormalized and, since the single particle wave functions are harmonic oscillators, the center-of-mass (CM) is removed.

One can generate the CM spurious states according to Ref. [13] and evaluate the overlap between these states and the nuclear eigenstates of the model (see Appendix B). Model components having with the corresponding CM components an overlap greater than 10% are treated as spurious states and discarded. The single-particle energies of these levels are taken from the known experimental level spectra of the neighboring nuclei [14]. For the particle-particle interaction, we use the G-matrix obtained from Yale potential [11]. These matrix elements are evaluated by applying the e^S correlation operator, truncated at the second order term of the expansion, to the harmonic oscillator base with size parameter b=1.76 fm. As elucidate in Refs. [3] and [4] the effective two-body potential used by the DCM and the BDCM models is separated in low and high momentum components. Therefore, the effective model matrix elements calculated within the present separation method and those calculated by Kuo [9] in the v_{low-k} approximation are pretty similar. The adopted separation method and the v_{low-k} generate two-body matrix elements which are almost independent from the radial shape of the different potentials generally used in structure calculations.
The particle-hole matrix elements could be calculated from the particle-particle matrix elements via a re-coupling transformation. We prefer to use the phenomenological potential of Ref. [15]. The same size parameter as for the particle-particle matrix elements has been used. In this contribution we present application of the S_n correlated model to the charge distribution of ^{6}He and to the electromagnetic transitions of neutron rich Carbon and Oxygen isotopes. In Fig. 2) Right three distributions are given for ^{6}He: 1) the correlated charge distribution calculated with the full S_3 operator, 2) the correlated charge distribution calculated with the partial S_3 operator obtained by neglecting the folded diagrams, 3) the charge distribution calculated for two correlated protons in the $1s_{\frac{1}{2}}$ shell. The full S_3 correlation operator therefore increases the calculated radii. The results obtained for the Carbon and Oxygen isotopes are in the following presented as function of the increasing valence neutrons. Before presenting the results is however worthwhile to remark that the high order correlation operators generate the interaction of the valence particles with the closed shell nucleus. The correlation model treats therefore consistently the "A" particles of the isotopes. By using generalized linearization approximations and cluster factorization coefficients we can perform exact calculations. In following Tables an over all b=1.76 fm has been used.

In Table 1, 3) we give the calculated magnetic moments and rms radii for one-hole and for one-particle in ^{16}O. The energy splitting between the ground- and the second (first) excited states and the electromagnetic transitions for the two isotopes are given in Tables 2, 4).

	DCM	Exp. [16]
Magnetic Moment (nm)	-1.88	-1.89
	DCM	Exp. [17]
rms (fm)	2.73	2.72(3)

Table 3: Magnetic moment (nm) and rms (fm) of the ground state of ^{17}O with $J = \frac{5}{2}^+; T = \frac{1}{2}$

Energy (MeV)	DCM	Exp. [16]
$\Delta E_{\frac{1}{2}^+\frac{5}{2}^+}$	0.87	0.89
Transition(e^2fm^4)	DCM	Exp. [16]
$BE(E2; \frac{1}{2}^+ \rightarrow \frac{5}{2}^+)$	2.10	2.18±0.16

Table 4: Energy splitting between the ground- and the first excited states and the E_2 transition for ^{17}O.

In Table 5) we give the calculated results for the energy splitting between the ground- and the 2^+ excited state and the corresponding electromagnetic transition for the ^{14}C with T=1. The commutator equations solved are given in Sec. 4.1).

In Tables 6, 7) we give the results for the energy splitting between the ground- and the excited states and the corresponding electromagnetic transitions for the ^{15}C and ^{19}O with T=$\frac{3}{2}$. The commutator equations used are given in Sec. 4.2). The resulting CMWFs are therefore including (3p) interacting with (4p1h). In Tables 8, 9) we give the results for the energy splitting between the ground- and the excited states and the corresponding E_2 transitions for the ^{16}C and ^{20}O with T=2 .Calculations are performed by extending the commutator equations given in Sec. 4.2) to four valence neutrons. The resulting CMWFs are then including (4p) interacting with (5p1h). Good results have been overall obtained with a neutron effective charge varying between 0.1- to 0.12-e_n.

6 Conclusion and Outlook

In this contribution we have investigated the effect of the microscopic correlation operators on the exotic structure of the Carbon and Oxygen isotopes. The microscopic correlation has been separated in short- and long-range correlations according to the definition of Shakin. The short-range correlation has been used to define the effective Hamiltonian of the model while the long-range correlation is used to calculate the structures and the distributions of exotic nuclei. As given in the work of Shakin, only the two-body short-range correlation need to be considered in order to derive the effective Hamiltonian especially if the correlation is of very short range. For the long range correlation operator the three body component is important and should not be neglected. Within the three body correlation operator, one introduces in the theory a three body interaction which compensates for the use of the genuine three body interaction of the no-core shell model. Within the S_2 effective Hamiltonian, good results have been obtained for the ground state energies and the distributions of ^{3}H, ^{3}He, and ^{4}He. The higher order correlation operators $S = 3 \cdots n$ have been used to calculate the structure and the electromagnetic transitions of ground

Energy (MeV)	Ref. [18]	BDCM	Exp. [19]
$\Delta E_{0^+2^+}$		8.38	8.32
Transition(e^2fm^4)	Ref. [18]	BDCM	Exp. [19]
$BE(E2; 2^+ \rightarrow 0^+)$	3.38	3.65	$3.74 \pm .50$

Table 5: Calculated energy splitting and $BE(E2; 2^+ \rightarrow 0^+)$ transition for ^{14}C.

Energy(MeV)	DCM	Exp. [20]
$\Delta E_{\frac{1}{2}^- \frac{1}{2}^+}$	3.15	3.10
Transition (e^2fm^2)	DCM	Exp. [20]
$BE(E1; \frac{1}{2}^- \rightarrow \frac{1}{2}^+)$	.014	.018

Table 6: Calculated energy splitting and $BE(E1; \frac{1}{2}^- \rightarrow \frac{1}{2}^+)$ transition for ^{15}C.

Energy (MeV)	Kuo [21]	DCM	Exp. [22]
$\Delta E_{\frac{5}{2}^+ \frac{1}{2}^+}$ (MeV)	-	1.45	1.47
Transition (W.U.)	Kuo [21]	DCM	Exp. [22]
$BE(E2; \frac{1}{2}^+ \rightarrow \frac{5}{2}^+)$	.39	.49	$.58 \pm .12$

Table 7: Calculated energy splitting and $BE(E2; \frac{1}{2}^+ \rightarrow \frac{5}{2}^+)$ transition for ^{19}O.

and first excited states for the isotopes of Carbon and Oxygen. By using generalized linearization approximations and cluster factorization coefficients we can perform expedite and exact calculations. Detailed calculations for the 5- and 6-neutron systems are presently under investigation and will be shortly reported.

A Definition of the Model CMWFs

A.1 CMWFs for Two "dressed" Particles

In the BDCM the degree of linearization applied to the commutator equations defines the CMWFs of the model. For A=6 the model space is formed by two valence particle states and by the full set of the (3p1h) CMWFs. These different components are associated to the following linearization mechanisms:
a) In the zero order linearization approximation we retain only two particle states:

$$\Psi^{2p}(j_1 j_2 J) = [a^\dagger_{j_1} a^\dagger_{j_2}]^{JM}|0\rangle \tag{16}$$

For the two particles we distinguish between :
1) effective valence space which is used to diagonalize the EoM,
2) complementary high excited single particle states which are used to compute the G matrix.
b) In the first order linearization approximation we include in the dynamic theory also the (3p1h) terms. These are generated by the application of the correlation operator of the third order to the particles in the open shell states. Within this linearization approximation the CMWFs of the model are defined by:

$$\Psi^{dressed}(j_1 j_2 J) = ([a^\dagger_{j_1} a^\dagger_{j_2}] + [a^\dagger_{j_1} a^\dagger_{j_2}]^{J_{12}} [a^\dagger_{j_3} a_{j_4}]^{J_{34}})^{JM}|0\rangle. \tag{17}$$

The (3p1h) CMWFs are then expanded according to the CFT theory. The expansion allows to orthonormalize the CMWFs in an easy way.
c) The (4p2h) states which characterize the second order linearization step are not included in the model space but, linearized, generate the eigenvalue equation of the model (2p)+(3p1h) states.

A.2 CMWFs for n "dressed' Particles

The CMWFs for n dressed particles are characterized by the coupling of the n valence particles with the (np(mp-mh)) core excited states. For both components we introduce cluster transformation coefficients (CFC) obtained within the CFT theory. With the use of these coefficients the complex base can be

Energy (MeV)	Ref. [18]	BDCM	Exp. [23]
$\Delta E_{0^+2^+}$	1.65	1.80	1.77
Transition (e^2fm^4)	Ref. [18]	BDCM	Exp. [23]
$BE(E2;2^+ \rightarrow 0^+)$	.85	.65	$.63^{\pm.11(stat)}_{\pm.16(syst)}$

Table 8: Calculated energy splitting and $BE(E2;2^+ \rightarrow 0^+)$ transition for ^{16}C.

Energy (MeV)	BDCM	Exp. [24]
$\Delta E_{0^+2^+}$	1.45	1.47
Transition (e^2fm^4)	BDCM	Exp. [24]
$BE(E2;0^+ \rightarrow 2^+)$	29.3	28.

Table 9: Calculated energy splitting and $BE(E2;0^+ \rightarrow 2^+)$ transition for ^{20}O.

easily orthogonalized and the CM can be eliminated. The numerical formulation of these coefficients will be published in short time.

B Center-of-mass Correction

Before performing the diagonalization of relative Hamilton's operator in the CMWFs defined in appendix A) we have to eliminate the spurious center-of-mass states. We start to compute, following the calculations of Ref. [13], the percent weights of spurious states in the model wave functions. These can be obtained by calculating the energy of the center of mass according to the following equation:

$$\begin{aligned} E_R = & \int dR \Psi^{\dagger dressed}(j_i j_j J)(R^2)\Psi^{dressed}(j_i j_j J) \\ & +2\sum_{ij}\int d\vec{r_i} d\vec{r_j} \Psi^{\dagger dressed}(j_i j_j J)(\vec{r_i}\cdot\vec{r_j})\Psi^{dressed}(j_i' j_j' J). \end{aligned} \tag{18}$$

In Eq. (18) the calculation of the integrals can be performed by expanding the dressed states in terms of the CFC given in [5] and by considering that for two particle states we have:

$$\begin{aligned} & \langle j_i j_j J|(\vec{r_i}\cdot\vec{r_j})|j_i j_j J\rangle \\ & = \tfrac{4\pi}{3}[\hat{j_i}\hat{j_j}] \begin{pmatrix} j_i & 1 & j_j \\ -\frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix}^2 \begin{Bmatrix} j_i & j_j & J \\ j_i & j_j & 1 \end{Bmatrix} \langle l_i|r|l_j\rangle^2, \end{aligned} \tag{19}$$

where:

$$\hat{j} = (2j+1). \tag{20}$$

By diagonalizing the above operator in the model space we obtain the energy of the center of mass. The overlap with the model space give the degree of "spuriosity" of the different components.

C The Correlated Operators

In this appendix we give the expectation values of operators calculated with dressed (correlated) particles. In the following we calculate the correlated distributions of even nuclei and the magnetic moment operator of odd nuclei. The distribution of two valence particles is evaluated from the model CMWFs:

$$\langle\tilde{\Psi}_{12}|\rho(r)|\tilde{\Psi}_{12}\rangle = \sum_{ij}\chi_{ij}\langle\Psi_{ij}|\rho(r)|\Psi_{ij}\rangle + \sum_{ijkl}\chi_{ijkl}\langle\Psi_{ij}\Psi_{kl}|\rho(r)|\Psi_{ij}\Psi_{kl}\rangle \tag{21}$$

For the magnetic moment operator we have also to calculate matrix elements between correlated CMWFs. Here for odd nuclei with one valence particle we calculate by using the corresponding correlated wave functions:

$$\langle\tilde{\Psi}^J|\mu|\tilde{\Psi}^J\rangle = \chi_{j_1}\langle\Phi_{j_1}|\mu|\Phi_{j_1}\rangle + \sum_{ljk}\chi_{ijk}\langle\Phi_{j_l}\Psi_{jk}|\mu|\Phi_{j_l}\Psi_{jk}\rangle \tag{22}$$

References

[1] F. Villars, *Proc. Enrico Fermi Int. School of Physics XXII*, Academic Press N.Y. (1961).

[2] C.M. Shakin and Y.R. Waghmare, *Phys. Rev. Lett.* **16** (1966), 403; C.M. Shakin, Y.R. Waghmare, and M.H. Hull, *Phys. Rev.* **161** (1967), 1006.

[3] M. Tomaselli, L.C. Liu, S. Fritzsche, T. Kühl, and D. Ursescu, *Nucl. Phys.* **A738** (2004), 216; M. Tomaselli, Ann. Phys. **205** (1991), 362.

[4] M. Tomaselli, L.C. Liu, S. Fritzsche, T. Kühl, *J. Phys. G: Nucl. Part. Phys.* **30** (2004), 999.

[5] M. Tomaselli, T. Kühl, D. Ursescu, and S. Fritzsche, Prog. Theor. Phys. **116** (2006), 699.

[6] K.A. Brückner, *The Many Body Problem*, John Wiley and Sons, New York (1959).

[7] T.T.S. Kuo and E. Osnes, *Folded-Diagrams Theory of the Effective Interaction in Atomic Nuclei*, Springer Lecture Notes in Physics Vol. 366, Berlin (1991).

[8] S.A. Moszkowski and B.L. Scott, Ann. of Phys. (N.Y.) **11** (1960), 55.

[9] M.F. Jiang, R. Machleidt, D.B. Stout, and T.T.S. Kuo, Phys. Rev. **C46**, 910 (1992); M. Lacombe, B. Loiseau J.M. Richard, R. Vinh Mau, J. Côté, P. Pirès, and R. de Tourreil, Phys. Rev. **C21** , 861 (1980); R. Machleidt, Adv. Nucl. Phys. **19** (1989), 189; S. Bogner, T.T.S. Kuo, L. Coraggio, A. Covello, and N. Itaco, Phys. Rev. **C65** (2002), 051301(R).

[10] K.E. Lassila, M.H. Hull, H.M. Ruppel, F.A.McDonald, and G. Breit, Phys. Rev. **126** (1962), 881.

[11] C.M. Shakin, Y.R. Waghmare, M. Tomaselli, and, M.H. Hull, Phys. Rev. **161** (1967), 1015.

[12] R. Machleidt, Adv. Nucl. Phys. **19**, 189 (1989).

[13] E. Baranger and C.W. Lee, Nucl. Phys. **22** (1961), 157.

[14] B.A. Brown, Prog. Part. Nucl. Phys. **47** (2001), 524.

[15] D.J. Millener and D. Kurath, Nucl. Phys. **255** (1975), 315.

[16] http://www.tunl.duke.edu/nucldata, and references therein quoted.

[17] C.W. de Jager, H. de Vries, and C. de Vries, Atomic Data and Nuclear Data Tables **14** (1974), 479.

[18] S. Fujii, T. Misuzaki, T. Otsuka, T. Sebe, and A. Arima, arXiv: nucl-th/0602002 (2006).

[19] S. Raman *et al.* Atomic Data and Nuclear Data Tables **31** (1987), 1.

[20] S. Fujii, R. Okamoto, and K. Suzuki, J. Phys.: Conf. Ser. **20** (2005), 83; arXiv :nucl.th/0505076 (2005).

[21] T.T.S. Kuo and E Osnes, J. Phys.: Nucl. Phys. **6** (1980), 335.

[22] F. Ajzenberg-Selove, Nucl. Phys. **A300** (1978), 1.

[23] N. Imai, H. J. Ong, N. Aoi, *et al.*, Phys. Rev. Lett. **92** (2004), 062501.

[24] J.K. Jewel, P.D. Cottle, K.W. Kemper, *et al.*,
$http://www.physics.fsu.edu/Nuclear/Reports/2001/PROG_O.pdf$.